WAVE ON STRING

Note: (*) → Multiple Correct Type Question

1. In a stationary wave **[NSEP-2018]**

(A) All the medium particles vibrate in the same phase.

(B) All The particles between two consecutive nodes vibrate in the same phase.

(C) Any two consecutive nodes vibrate in the same phase.

(D) All the particles between two consecutive antinodes vibrate in the same phase.

Ans. (B)

Sol. All the particles between two consecutive nodes vibrate in same phase.

2. A student while performing an experiment with a sonometer with bridges separated by a distance of 80 cm , missed out some of the observations. However, he claimed that the three resonant frequencies for a given tuning fork were 84,140 and 224 Hz . The speed of transverse waves on the wire is **[NSEP-2018]**

(A) 33.30 m / s (B) 330.0 m / s (C) 44.80 m / s (D) 448.0 m / s

Ans. (C)

Sol. f = fundamental frequency

$n_1{}^{th}$ harmonic $= 84$

$n_2{}^{th}$ harmonic $= 140$

$n_3{}^{th}$ harmonic $= 224$

$n_1 f : n_2 f : n_3 f = 84 : 140 : 224$

$n_1 = 3$

$n_2 = 5$

$n_3 = 8$

$n_2 f - n_1 f = 140 - 84$

$(5-3)f = 56$

$f = 28$ Hz

$28 = \frac{V}{2\ell}$

$v = 44.80$ m / s

3. Two wires, made of same material, one thick and the other thin are joined to form one composite wire. The composite wire is subjected to the same tension throughout. A wave travels along the wire and passes the point where the two wires are joined. The quantity which changes at the joint are **[NSEP-2019]**

(A) Frequency only.

(B) Propagation speed only.

(C) Wavelength only.

(D) Both propagation speed and wavelength.

Ans. (D)

***4.** A rope of mass m and length L is suspended vertically. A mass M is suspended from bottom of the rope. A transverse wave is produced on the rope, which travels the length of rope in time t choose the CORRECT statement(s) **[NSEP-2022]**

(A) $t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m}-\sqrt{m}\right)$

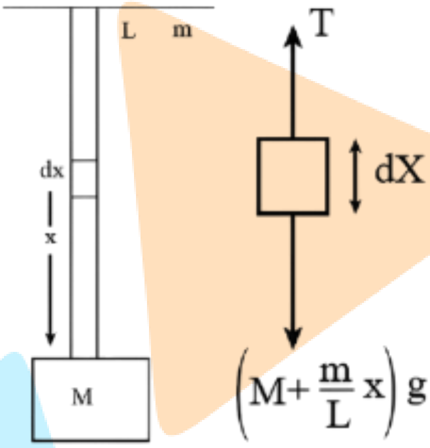
(B) for $m \ll M$, the time $t = \sqrt{\frac{mL}{Mg}}$

(C) For $M = 0$ (i.e., no mass hanging), the time, $t = \sqrt{\frac{L}{g}}$

(D) Time to travel the lower half of the rope by the wave is less than that to travel the upper half

Ans. (AB)

Sol. Considering a small element dx at a distance x from M,



$$T = \left(M + \frac{m}{L}x\right)g$$

Velocity of the wave in the string is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(M + \frac{m}{L}x\right)g}{\frac{m}{L}}}$$

$$v = \sqrt{\left(\frac{ML}{m} + x\right)g}$$

$$\frac{dx}{dt} = \sqrt{\left(\frac{ML}{m} + x\right)g}$$

$$\int_0^L \frac{dx}{\sqrt{\left(\frac{ML}{m} + x\right)}} = \sqrt{g}\int_0^t dt$$

$$\left[2\sqrt{\frac{ML}{m} + x}\right]_0^L = \sqrt{g}t$$

$$\left[2\sqrt{\frac{ML}{m} + L} - 2\sqrt{\frac{ML}{m}}\right] = \sqrt{g}t$$

$$\Rightarrow t = 2\sqrt{\frac{L}{mg}}\left[\sqrt{M+m} - \sqrt{M}\right]$$

$\therefore$ Option A is correct

For $m \ll M$

$$t = 2\sqrt{\frac{L}{mg}}\sqrt{M}\left[\left(1+\frac{m}{M}\right)^{\frac{1}{2}} - 1\right]$$

$$t = 2\sqrt{\frac{L}{mg}}\sqrt{M}\left[1+\frac{m}{2M} - 1\right]$$

$$\Rightarrow t = \sqrt{\frac{mL}{Mg}}$$

Option B is correct

For $M = 0$,

$$t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)$$

$$t = 2\sqrt{\frac{L}{g}}$$

$\therefore$ Option C is incorrect.

In the lower half of the string tension is lesser, so, speed is lesser, and it will take more time to travel through the lower half compared to upper half.

$\therefore$ Option D is incorrect.

5. A rope of mass M and length L hangs vertically. Time needed for a transverse pulse to travel from its bottom end to the support is **[NSEP-2023]**

(A) $\sqrt{\frac{2L}{g}}$ (B) $2\sqrt{\frac{L}{g}}$ (C) $\sqrt{\frac{L}{g}}$ (D) $\sqrt{\frac{L}{2g}}$

Ans. (B)

Sol. $V_\omega = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg \times L}{L \times M}} = \sqrt{gx}$

$$V^2 = gx \Rightarrow 2V \cdot \frac{dV}{dx} = g$$

$$\Rightarrow a = \frac{g}{2}$$

$$L = \frac{1}{2} \times \frac{g}{2} \times t^2$$

$$t^2 = \frac{4L}{g} \Rightarrow t = \sqrt{\frac{4L}{g}} = 2\sqrt{\frac{L}{g}}$$

6. Which of the following functions does not represent a traveling wave **[NSEP-2023]**

(A) $y = A\sin^2\left[\pi\left(t - \frac{x}{v}\right)\right]$ (B) $y = Ae^{-\alpha t}\cos(kx - \omega t)$

(C) $y = A\sin\left[(kx)^2 - (\omega t)^2\right]$ (D) $y = A\cos\left[(kx - \omega t)^2\right]$

Ans. (C)

Sol. $F(ax+bt)$

***7.** Two plane progressive waves travelling on a string as **[NSEP-2023]**

$y_1 = 2.5\times10^{-3}\sin(30x-420t)$

$y_2 = 2.5\times10^{-3}\sin(30x+420t)$

superimpose to produce a standing wave. The variables x and y are in meter and t is in second. Then

(A) The equation of resultant standing wave is $y = 5\times10^{-3}\cos(30x)\sin(420t)$

(B) The equation of resultant standing wave is $y = 2.5\times10^{-3}\sin(30x)\cos(420t)$

(C) The antinode closest to $x = 0.25\text{m}$ is at $x = 0.262\text{m}$

(D) The amplitude of oscillation of particle at $x = 0.17$ m is 4.63 mm

Ans. (CD)

Sol. (A) & (B)

$y_{net} = y_1 + y_2$

$= 2.5\times10^{-3} = [\sin(kx-\omega t)+\sin(k'x+\omega t)]$

$= 2.5\times10^{-3}2\sin kx\cos\omega t$

$y_{net} = 5\times10^{-3}\sin(30x)\cos(420t)$

Both option are wrong

(C) For antinode

$\sin 30x = \pm1$

$30x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\pi}{60}$

$x = \frac{\pi}{60}, \frac{3\pi}{60}, \frac{5\pi}{60}, \frac{7\pi}{60}, \ldots\ldots\ldots\ldots$

$x = 0.0523, 0.157, 0.262, 0.366$

nearest antinode to $x = 0.25$ is

$x = 0.262$

(D) Amplitude $= 5\times10^{-3}\sin 30x$

for $x = 0.17$

$A = \left|5\times10^{-3}\sin(30\times0.17)\right|$

$A = 4.63\times10^{-3}$ m

$A = 4.63$ mm

SOUND WAVE

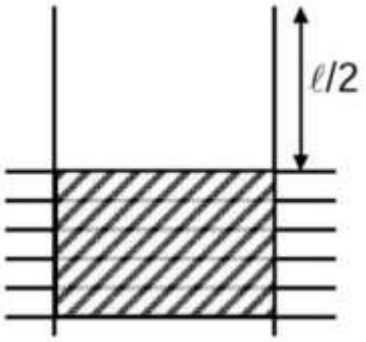
Note: (*) → Multiple Correct Type Question

1. A whistle whose air column is open at both ends has a fundamental frequency 500 Hz . The whistle is dipped in water such that half of it remains out of water. What will be the fundamental frequency now? (speed of sound in air is $340\ ms^{-1}$) **[NSEP-2017]**

(A) 250 Hz (B) 125 Hz (C) 500 Hz (D) 1000 Hz

Ans. (C)

Sol. $\frac{v}{\frac{4\ell}{2}} = \frac{v}{2\ell} = 500\ \text{Hz}$



2. A man stands at rest in front of a large wall. A sound source of frequency 400 Hz is placed between him and the wall. The source is now moved towards the wall at a speed of $1\ m/s$. The number of beats heard per second will be(speed of sound in air is $345\ m/s$) **[NSEP-2017]**

(A) 0.8 (B) 0.58 (C) 1.16 (D) 2.32

Ans. (D)

Sol. $\Delta f = \frac{c}{c-u}\cdot f - \frac{c}{c+u} f = \frac{cf\,2u}{c^2-u^2} = \frac{2uf}{c}$

$\Delta f = \frac{2\times 1\times 400}{345} = \frac{800}{345} = 2.32$

3. An empty earthen pitcher is kept under a water tap and starts filling with water as the tap is opened. The pitch of the sound produced **[NSEP-2018]**

(A) Goes on decreasing

(B) Goes on increasing

(C) First increases and then decreases after the pitcher is half filled

(D) Does not change

Ans. (B)

Sol. Since frequency $\propto \frac{1}{\ell}$ (here ℓ is empty space)

$\Rightarrow$ with time ℓ decreases and frequency increases so pitch also increases

4. A sound source of constant frequency travels with a constant velocity past an observer. When it crosses the observer the sound frequency sensed by the observer changes from 449 Hz to 422 Hz . If the velocity of sound is $340\ m/s$, the velocity of the source of sound is **[NSEP-2019]**

(A) $8.5\ m/s$ (B) $10.5\ m/s$ (C) $12.5\ m/s$ (D) $14.5\ m/s$

Ans. (B)

Sol. $$\frac{C}{\frac{C-v}{C}}f=449$$

$$(C+v)422=(C-v)449$$

$$v=\frac{27C}{(422+449)}=\frac{27\times 340}{871}=10.53\ m/s$$

5. Two factories are sounding their sirens at 400 Hz each. A man walks from one factory towards the other at a speed of $2\ m/s$. The velocity of sound is $320\ m/s$. The number of beats heard by the person in one second will be: **[NSEP-2019]**

(A) 6 (B) 5 (C) 4 (D) 2.5

Ans. (B)

Sol. $$If=\left(\frac{C+v}{C}-\frac{C-v}{C}\right)f$$

$$=\frac{2V}{C}f=\frac{4}{320}\times 400=5$$

6. The frequency of the third overtone of a closed end organ pipe equals the frequency of the fifth harmonic of an open end organ pipe. Ignoring end correction, the ratio of their lengths $l_{open}:l_{close}$ is **[NSEP-2019]**

(A) $10:7$ (B) $10:9$ (C) $2:1$ (D) $7:10$

Ans. (A)

Sol. $$\frac{7\ V}{4\ L_{close}}=\frac{5\ V}{2\ L_{open}}\Rightarrow\frac{L_{open}}{L_{close}}=\frac{10}{7}$$

***7.** The intensity of sound at a point P is I_0, when the sounds reach this point directly and in same phase from two identical sources S_1 and S_2. The power of S_1 is now reduced by 64% and the phase difference (ϕ) between S_1 and S_2 is varied continuously. The maximum and minimum intensities recorded at P are now I_{max} and I_{min} such that **[NSEP-2020]**

(A) $I_{max}=0.64I_0$ (B) $I_{min}=0.36I_0$ (C) $\frac{I_{max}}{I_{min}}=16$ (D) $\frac{I_{max}}{I_{min}}=\frac{16}{9}$

Ans. (AC)

Sol. The resultant intensity is $I_0=I+I+2\sqrt{I\times I}\cos 0=4I$ when the intensity of one source is reduced by 64% it becomes $I-0.64I=0.36I$ Then the resultant intensity becomes

$I_{Result}=0.36I+I+2\sqrt{0.36I\times I}\cos\phi=I_0(0.34+0.30\cos\phi)$ where phase ϕ is now varied.

When $\phi=0$, The intensity at P is $I_{Result}=0.64I_0=I_{max}$ Ans a

When $\phi=\frac{\pi}{2}$. The intensity at P is $I_{Result}=0.34I_0$

When $\phi=\pi$ The intensity at P is $I_{Result}=0.04I_0=I_{min}$

This shows that $\frac{I_{max}}{I_{min}}=\frac{0.64}{0.04}=16$ Ans c

8. A cycle wheel of mass M and radius R fitted with a siren at a point on its circumference, is mounted with its plane vertical on a horizontal axle at about 3 feet above the ground. An observer stands in the vertical plane of the wheel at 100 m away from the axle of the wheel on a horizontal platform. The siren emits a sound of frequency 1000 Hz and the wheel rotates clockwise with a uniform angular speed $\omega = \pi \text{rad}/\sec$. Initially at $t = 0\sec$ the siren is nearest to the observer and moves downwards. The observer records the highest pitch of sound for the first time after (speed of sound in air is 330 ms^{-1}) **[NSEP-2021]**

(A) 0.30 s (B) 1.8 s (C) 2.3 s (D) 9.8 s

Ans. (B)

Sol. The observer will record the maximum frequency $v' = \frac{v}{v - v_s} \times v$ when the sound produced by the siren, in the top most point of the circumference of the wheel, reaches the listener. This sound will reach the listener in time $t = \frac{100}{330} = 0.303 \text{ s}$ after being produced. Also the wheel itself will take time

$t_0 = \frac{3}{4} \times \frac{2\pi}{\omega}$ substituting $\omega = \pi$

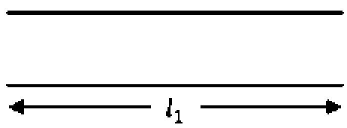
we get $t_0 = \frac{3\pi}{2\pi} = 1.5 \text{ s}$ Hence the total time is $t + t_0 = 0.303 + 1.5 = 1.803 \text{ s}$

9. A sound source of fix frequency is in unison with an open end organ pipe of length 30.0 cm and a close end organ pipe of length 23.0 cm (both of same diameter). Both pipes are sounding their first overtone. If velocity of sound is 340 ms^{-1}, frequency of sound source is nearly **[NSEP-2022]**

(A) 1000 Hz (B) 1062 Hz (C) 1100 Hz (D) 1018 Hz

Ans. (B)

Sol. For open end organ pipe:



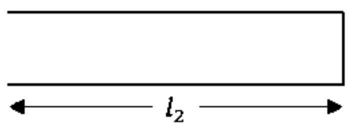
First overtone frequency, $f_1 = \frac{2v}{2l_1}$

For closed organ pipe:

First overtone, $f_2 = \frac{3v}{4l_2}$

According to question, $\frac{2v}{2l_1} = \frac{3v}{4l_2} \Rightarrow \frac{l_1}{l_2} = \frac{4}{3}$

Considering end correction, $\frac{l_1}{l_2} = \frac{30 + (2 \times (0.6r))}{23 + 0.6r}$



$\therefore$ from (1) & (2) $\Rightarrow r = \frac{2}{1.2}$

$\therefore$ end correction: $0.6r = 0.6 \times \frac{2}{1.2} = 1 \text{ cm}$

$\therefore$ frequency, $f_1 = \frac{2v}{2 \times (l_1)} = \frac{340}{0.32} = 1062.5 \text{ Hz}$

$f_2 = \frac{3}{4}\left(\frac{v}{l_2}\right) = \frac{3}{4} \times \frac{340}{0.24} = 1062.5 \text{ Hz}$

10. A police car, moving at speed of 108 km/h hour, approaches a truck moving at 72 km/h hour in opposite direction. The natural frequency of the siren of the car is 800 Hz and the surrounding temperature is $27^\circ C$. The frequency heard by the truck driver as the car passes him

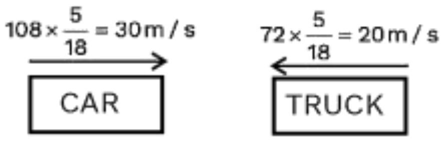
[NSEP-2023]

(A) Remains unchanged
(B) Decreases nearly by 232 Hz
(C) Increases nearly by 231 Hz
(D) Decreases nearly by 260 Hz

Ans. (B)

Sol. Speed of sound in air at $27^\circ C$ is 347 m/s

Initially,



$F_0 = 800$ Hz

$$F_{ap} = 800\left[\frac{C+20}{C-30}\right] = 926$$

Finally

$$F'_{ap} = 800\left[\frac{C-20}{C+30}\right] = 693$$

So, frequency decreases by 232 Hz .

11. When the speaker S_1 is switched ON, the sound intensity at a point P in a room is 80dB. But when the speaker S_2 is switched ON(S_1 is switched OFF), the sound intensity at the same point P in the room is 85 dB. The sound intensity level (in dB) at the same point P in the room, if the two speakers S_1 and S_2 are simultaneously switched ON, is (consider the speakers to be incoherent)

[NSEP-2023]

(A) 165 dB (B) 86.2 dB (C) 87.8 dB (D) 88.6 dB

Ans. (B)

Sol. $80 = 10\log\frac{I_1}{I_0}$

$I_1 = (10)^8 \times 10^{-12} = 10^{-4}$

$I_2 = (10)^{8.5} \times 10^{-12} = 10^{-3.5}$

$L = 10\log\frac{(I_1 + I_2)}{I_0}$

12. An open-end organ pipe 30 cm in length and a closed-end organ pipe 23 cm in length, both of equal diameter, are each sounding their first overtone and both are in unison at 1100 Hz . The speed of sound in air, is estimated to be nearly **[NSEP-2023]**

(A) 324 ms^{-1} (B) 332 ms^{-1} (C) 340 ms^{-1} (D) 352 ms^{-1}

Ans. (D)

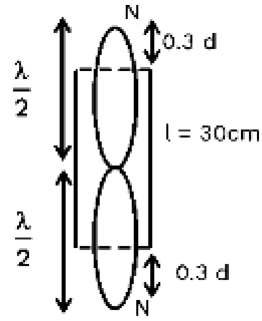
Sol. $\lambda = 0.30 + 0.6\ d$

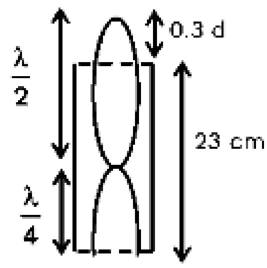
$v = f\lambda = f \times (0.30 + 0.6d)$ ... (i)

$\frac{3\lambda}{4} = 0.23 + 0.3d$

$\lambda = \frac{4}{3}(0.23 + 0.3d)$

$v = f.\frac{4}{3}(0.23 + 0.3d)$ ... (ii)





From (1) & (2)

$\frac{4}{3}(0.23 + 0.3d) = (0.30 + 0.6d)$

$0.6\ d = 0.02$

From (1) $\Rightarrow v = 1100 \times (0.30 + 0.02) = 352\ m/s$

ELECTROMAGNETIC WAVES

Note: (*) → Multiple Correct Type Question

1. In an electromagnetic wave the phase difference between electric vector and magnetic vector is **[NSEP-2019]**

(A) Zero (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$

Ans. (A)

Sol. There is no phase diff. between E and B

***2.** The electric field component of an electromagnetic wave is expressed as $E=\left(3\hat{j}+bk\right)\times10^{-3}\sin\left[10^{7}\left(x+2y+3z-\beta t\right)\right]$ in SI units. Taking $c=3\times10^{8}ms^{-1}$ as the speed of electromagnetic wave in vacuum, choose the correct option(s) **[NSEP-2021]**

(A) The value of constant beta is $\beta=3\times10^{8}\times\sqrt{14}$

(B) The value of constant b is $b=2$.

(C) The average energy density of the em wave is $U=6.5\times10^{-6}\varepsilon_0$ in SI units.

(D) The amplitude of magnetic field is $B=1.20\times10^{-11}$ Tesla

Ans. (ACD)

Sol. Given that the Electric field $\vec{E}=\left(3\hat{j}+bk\right)\times10^{-3}\sin\left[10^{7}\left(x+2y+3z-\beta t\right)\right]$

Knowing $\vec{K}\cdot\vec{r}=\left(\hat{i}k_x+\hat{j}k_y+\hat{k}\,k_z\right)\cdot\left(\hat{i}\,x+\hat{j}y+\hat{k}\,z\right)=xk_x+yk_y+zk_z$.

Comparing it with the given expression we get $xk_x+yk_y+zk_z=10^{+7}\left(x+2y+3z\right)$

Thereby $\Rightarrow k_x=10^{7},k_y=2\times10^{7}$ and $k_z=3\times10^{7}$ or the vector $\vec{k}=\left(\hat{i}+2\hat{j}+3k\right)\times10^{7}$

$\Rightarrow K=10^{7}\sqrt{14}$

Also the speed of the wave $c=\frac{\beta}{K}$ $\therefore\beta=c\times10^{7}\sqrt{14}=3\times10^{15}\sqrt{14}$

Further for any electromagnetic wave $\dot{k}\cdot\vec{E}=0$

Therefore $10^{+7}\left(\hat{i}+2\hat{j}+3k\right)\cdot\left(3\hat{i}+bk\right)\times10^{-3}=0\Rightarrow2\times3+3\,b=0\Rightarrow\;b=-2$

this makes option b wrong. Further the energy of an em wave is

$=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0\left(\sqrt{3^2+2^2}\right)^2\times10^{-6}=6.5\varepsilon_0\mu\,J$.

The magnetic field can be obtained as

$B=\frac{E}{c}=\frac{10^{-3}\sqrt{13}}{3\times10^{8}}=1.20\times10^{-11}$ Tesla

***3.** The magnetic field $\vec{B}=2\times10^{-5}\sin\left\{\pi\left(0.5\times10^{3}x+1.5\times10^{11}t\right)\right\}\hat{j}\;T$ represents a plane electromagnetic wave travelling in space with x in meter and t in second. The correct statement(s) are **[NSEP-2023]**

(A) The wave length of the wave is 4.0 mm and its frequency is 75 GHz

(B) The energy density associated with the wave is nearly $=316\mu\,J/m^3$

(C) The electric field vector is $\vec{E}=-6000\sin\left[\pi\left(0.5\times10^{3}x-1.5\times10^{11}t\right)\right]kV_{m^{-1}}$

(D) The electric field vector is $\vec{E} = 6000\sin\left[\pi\left(0.5\times10^{3}x + 1.5\times10^{11}t\right)\right]\hat{k} \vee m^{-1}$

Ans. (AD)

Sol. $B_0 = 2\times10^{-5}$

$K = 0.5\pi\times10^{3}$

$\omega = 1.5\pi\times10^{11}$

$\vec{B} \to \hat{j}$, wave $\to -\hat{i}$

$\vec{E} = +\hat{K}$

(A) $\lambda = \dfrac{2\pi}{K} = 4$ mm

$f = \dfrac{\omega}{2\pi} = 75\text{GHz}$

(B) Energy density $= \dfrac{B_0^2}{2\mu_0} = \dfrac{\left(2\times10^{-5}\right)^2}{2\times4\pi\times10^{-7}}$

$= 159.2\mu\ J/m^3$

(C) & (d)

$E_0 = CB_0 = 3\times10^{8}\times2\times10^{-5}$

$E_0 = 6\times10^{3}$

Direction $= +\hat{k}$

(D) option is correct